

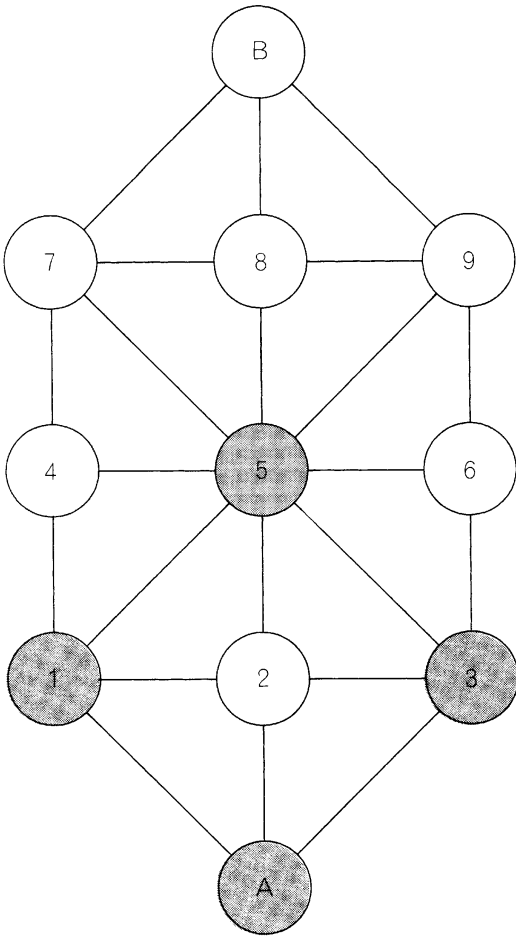
5. Four Unusual Board Games

DURING the 1960's there was a remarkable upsurge of interest in mathematical board games. Today more people than ever before are playing the traditional games such as chess and experimenting with the new games that keep turning up in the stores. More mathematicians are analyzing the strategies of such games and more computers are being programmed to play them. In this chapter we examine four excellent but little-known board games, two new and two old. Their playing fields can easily be drawn on paper or cardboard, the rules of play are quite simple and everyone in the family will find the contests great fun.

The Military Game, as it is called in France, is a splendid example of a two-player game that combines extreme simplicity with extraordinary strategic subtlety. According to Édouard Lucas, who describes the game in Volume III (pages 105–116) of his celebrated *Récréations Mathématiques*, the game was popular in French military circles during and after the Franco-Prussian War of 1870–1871. It is a

pity that it has since been so completely forgotten; not one of the standard histories of board games even mentions it.

The board for the Military Game is shown in Figure 30 with the positions labeled to facilitate description. One player—we will call him White—has three men that are initially placed on the colored spots. *A*, *I*, and *3*. Black, his opponent, has only one man, which he places on spot *5* in the center. (Chess pawns can be used for men, or three pennies and a nickel.) White moves first and the game proceeds with alternate turns. Black may move in any direction along a line from one spot to a neighboring spot. White moves similarly, but only left, right or forward (straight ahead or diagonally), never backward. There are no captures. White wins if he can pin Black's piece so that it cannot move. This usually occurs with Black on spot *B*, but it can also occur with Black on spot *4* or *6*. Any other outcome is a win for Black. He wins if he slips behind "enemy lines," making it impossible for White to pin him, or if a situa-



30. The French Military Game

tion develops in which the same moves are endlessly repeated.

The game is as simple to learn as ticktacktoe, but it is more exciting to play and more difficult to analyze. Lucas is able to show that White, if he plays rationally, can always win, but there is no simple strategy and the game abounds in traps and surprises. Often the best move is the move that seems to be the worst. An experienced Black has little

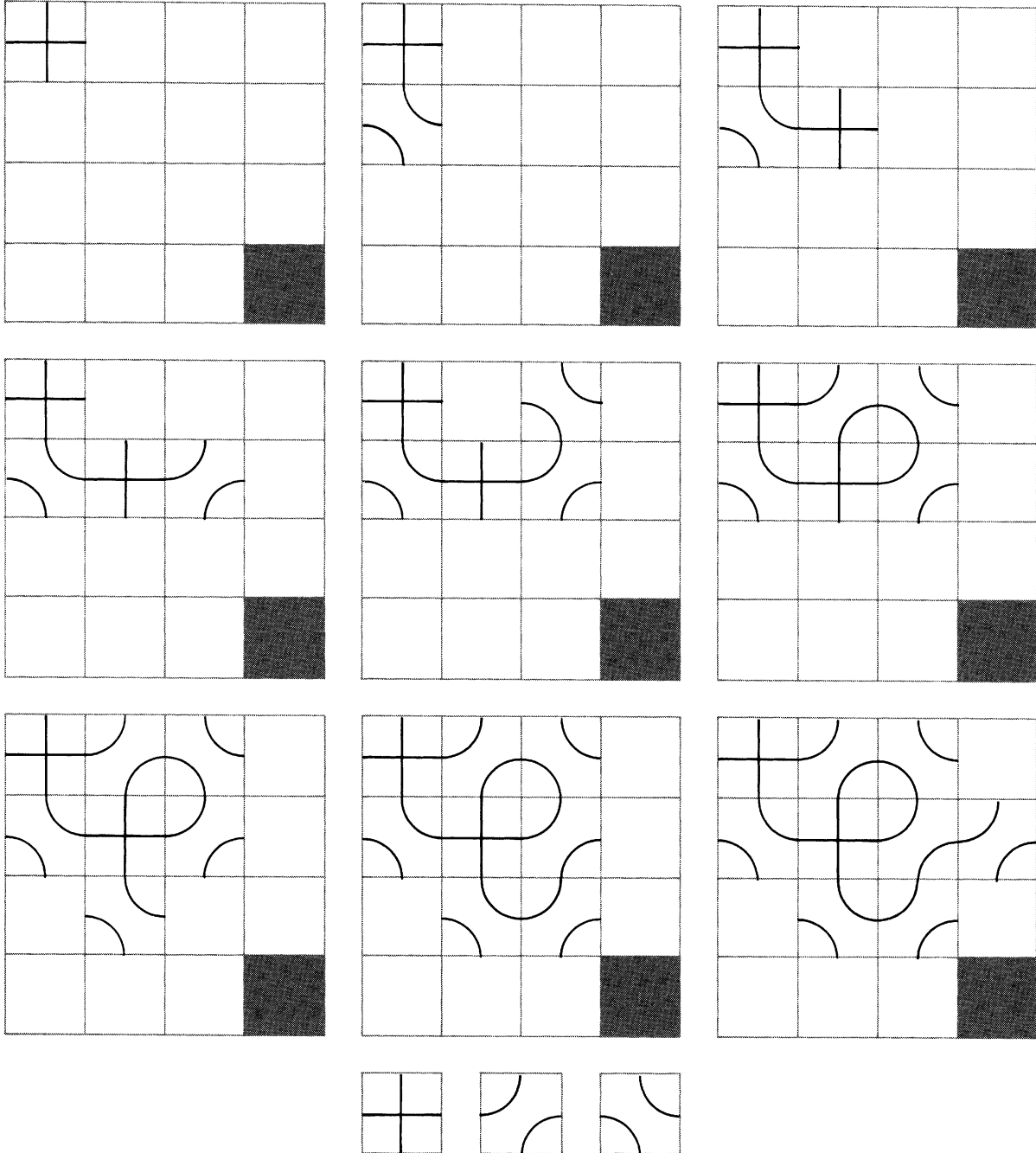
difficulty escaping from an inexperienced White.

Suppose we increase Black's freedom by permitting him to place his piece, at the start of the game, on *any* spot he chooses? Who now wins if both sides play rationally?

Topological board games, on which players construct paths that twist about over the field, are recent developments. Hex, Bridg-it, Zig-Zag, Roadblock, Pathfinder, Squirt, Twixt: these are trade names of some of the games of this type that have been marketed during the past thirty years. In 1960 William L. Black, then an undergraduate at the Massachusetts Institute of Technology, made a study of Hex and Bridg-it, two games discussed in earlier collections of my columns. An outcome of this study was a novel topological game his friends called Black.

Although marked tiles can be used, Black is easily played as a pencil-and-paper game on a checkered field. The size of the field is optional; the standard eight-by-eight field seems ideal, but it is simpler to explain the game on the smaller four-by-four. After the field is drawn the first player starts the game by making a cross in the upper left corner cell as shown in Figure 31. The second player continues the path by making one of three permissible marks in a cell adjacent to the first cell marked. The three marks, shown at the bottom of the illustration, are each composed of two lines. One line represents one of the three ways in which the path can be joined to an open side of the square; the second is added to connect the remaining two sides.

The players alternate moves. Each move



31. William L. Black's game

must extend the path into a neighboring cell. Each player tries to avoid running the path to a border of the field. If he is forced to carry the path to the border, he loses the game. He wins if he succeeds in extending the path into (not just to the border of) the lower right corner cell [*shown shaded*]. The illustration shows successive moves of a typical short game. The first player wins by forcing his opponent to play in the upper right corner cell, where any mark will carry the path to the edge of the field. (Note that the cross extends the path only along one of its arms, although the other arm may become part of the path as the result of a later play.)

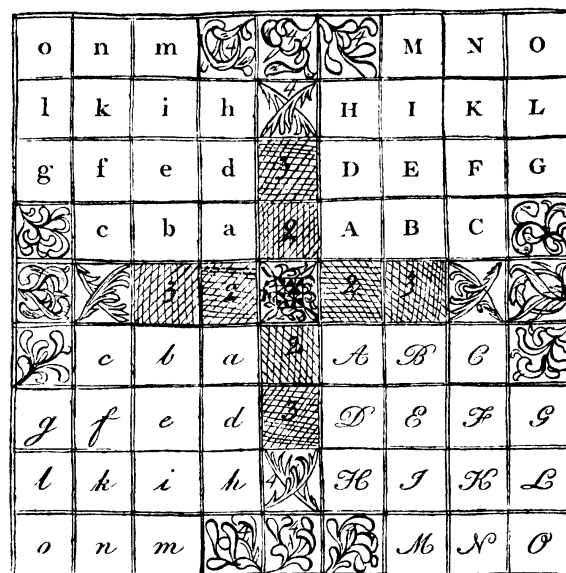
The game of Black is of special interest because soon after it was conceived a friend of Black's, Elwyn R. Berlekamp, hit on an elegant strategy that guarantees a win for one of the players. The strategy applies to rectangular fields of any size or shape. Since knowledge of the strategy destroys all interest in actual play, I urge you to play the game and see if you can match Berlekamp's brilliant insight before checking the answer section.

One of the best of many medieval board games is a game that seems to have been first played in Scandinavian countries as early as the fourth and fifth centuries, when it was called *tafl*. In later centuries it was known as *hnefatafl*. The Norsemen introduced the game to Britain, where it was the only board game played by the early Saxons until it began to be replaced by chess in the eleventh and twelfth centuries. H. J. R. Murray, in his *History of Board-Games*

Other than Chess, gives reasons for thinking that this is essentially the same game that was still being played in the sixteenth century in Wales, under the name of *tawlbwrdd*, and in the eighteenth century in Lapland, where it was known as *tablut*.

It was Murray who discovered that the great Swedish botanist Carolus Linnaeus included a full description of *tablut* in an extensive diary he kept during his exploration of Lapland in 1732. An English translation of the diary, by Sir James Edward Smith, was published in London in 1811 with the title *Lachesis Lapponica: or a Tour of Lapland*. Figure 32 is a reproduction of the *tablut* board as it is shown on page 55 of Volume II of that edition.

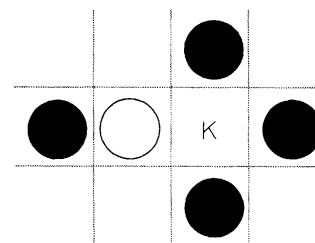
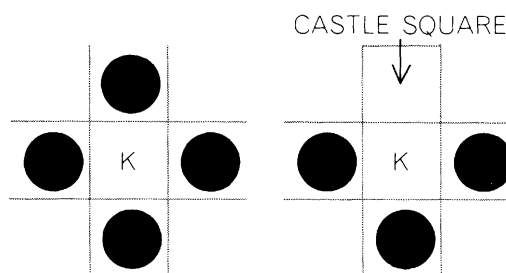
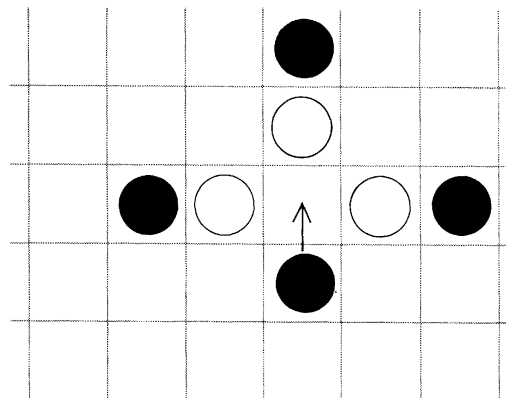
32. The game of tablut



White pieces, representing light-haired Swedes, include a single king and 8 warriors. Black pieces, 16 in number, represent Muscovite warriors. (It is convenient to use a white chess king and 8 white pawns for the Swedes. Black chessmen can be used for the Muscovites, but all must be regarded as identical pieces.) Each black and white piece, including the king, moves like a rook in chess, that is, an unlimited distance along vacant cells in a straight line paralleling a side of the board.

The game begins with the Swedish king occupying the center square, which is known as the castle. Only the king is permitted to stand in the castle, although any piece may move through it when it is vacant. Surrounding the king, on the 8 shaded squares, are his eight warriors. The Muscovites occupy the 16 decorated squares at the four sides of the board.

Either player may open the game. Enemy pieces are captured by a pincer move that consists of occupying adjacent cells on opposite sides of a piece, the three pieces being in the same row or column. For example, if Black makes the indicated move, he captures the three white pieces simultaneously [see Figure 33, top drawing]. If a piece moves between two enemy pieces, however, it is not captured by them. The king may take part in capturing enemy pieces, but he himself is captured only if he is surrounded on all four sides by four enemy pieces or by three enemy pieces and the castle square [middle drawing]; he cannot move from his castle into such a formation without being captured.



33. *Methods of capture in tablut*

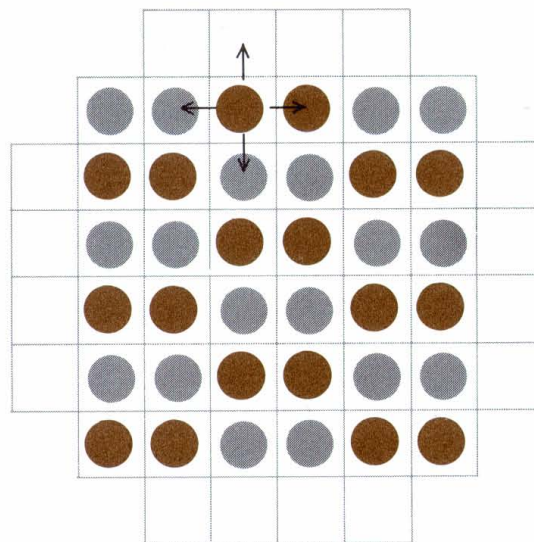
Linnaeus adds that when the king is in his castle, with three enemy warriors on three sides, and one of his own men on the fourth side, the Swedish warrior is taken if a Muscovite moves to the cell next to the Swede on the side opposite the king [bottom drawing].

Black's objective is to capture the king.

If this occurs, the Muscovites win. White's objective is to allow the king to flee the country by reaching any cell on the perimeter of the board. Whenever there is an unobstructed path along a row or column by which the king can reach the border, White must warn Black by saying "Raichi!" (a remark similar in function to "Check!" in chess). If there are two escape paths, White calls out "Tuichu!" Of course "Tuichu!" announces a win for White because there is no way Black can block two escape routes with a single move.

Sidney Sackson, a New York City engineer who makes a hobby of collecting board games, knows of only one occasion on which *tablut* has been made and sold in this country. In 1863 it was issued as a Civil War game called Freedom's Contest, or the Battle for the Union. This game is identical with *tablut* except that the king is called the "Rebel chief" and the pieces are Rebel and Union soldiers. The Rebel chief is limited to a maximum move of four spaces. The traditional game seems to favor White, so perhaps this restriction was intended to redress the balance. (*Breakthru*, Minnesota Mining and Manufacturing's Bookshelf Game currently on sale, is based on *tablut*.)

Sackson is himself the inventor of many unusual board games, one of the best of which he calls *Focus*. It is played with 36 counters, half of them one color and half another. Small poker chips of the interlocking variety make excellent pieces. They are placed initially on an eight-by-eight board from which three cells at each corner have



34. Sidney Sackson's game of *Focus*

been removed. Figure 34 shows how the pieces (black and colored in this case) are arranged.

Either side may move first. A move consists of moving a "pile" of pieces (at the outset all piles are one chip high) as many spaces as there are pieces in the pile. Moves are vertical or horizontal, never diagonal. The four possible moves of one colored piece at the start of a game are shown in Figure 34. If the piece moves up, it lands on a vacant square. A move to the right puts it on top of another colored piece, to the left or down puts it on top of a black piece. The last three moves form two-high piles. Such piles may be moved two spaces in any direction. Piles of three, four, and five pieces move three, four, and five spaces respectively. A pile is controlled by the player who owns the piece on top. In moving it does not matter whether the intervening cells are empty or occupied by piles controlled by either player. Passed-over pieces are not

affected in any way. A move may end on a vacant cell or on another pile. Figure 35 shows the possible moves of a two-high pile.

Piles may not contain more than five pieces. If a move produces a pile of more than five, all pieces in excess of five are taken from the bottom of the stack. If they are enemy pieces, they are considered captured and are removed from the game. If they belong to the player making the move, they are placed aside as reserves. At any time during the game a player may, if he wishes, take one of his reserve pieces and place it on any cell of the board, empty or otherwise. It has the same effect as a moved piece: if it goes on a pile, the pile belongs to the player who placed it. Using a reserve piece substitutes for a move on the board.

A player may, if he wishes, make a move of fewer spaces than the number of pieces in the pile being moved. He does this by taking from the *top* of the pile as many pieces as the number of spaces he wishes

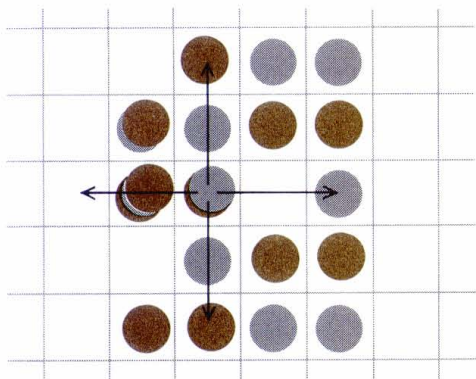
to move. The rest of the pieces stay where they are. For example, a player may take the top three pieces of a five-high pile and move them three spaces. The pile that *remains* after such a move belongs to the player who owns the piece on top.

When a player is unable to move (that is, controls no piles and has no reserves), the game is over and his opponent wins.

One additional rule is needed. As Paul Yearout, a mathematician at Brigham Young University, pointed out, the second player can always achieve at least a draw by symmetry play; that is, after each move by the first player, he duplicates the move by a symmetrically opposite play. To prevent this, Sackson suggests either of the following alternatives: (1) A draw is declared a win for the first player, (2) Before the game begins each player switches one of his pieces for one of his opponent's pieces (the second player must make an exchange that does not restore symmetry to the pattern) and the game then proceeds as described.

Focus was marketed by Whitman Publishing Company in 1965, the first of Sackson's many marketed games. For a more detailed account of the game as well as suggestions for strategic play, see pages 125–134 of Sackson's *Gamut of Games*.

35. Moves in the game of Focus



Answers

Which side wins the French Military Game if Black is given the privilege of starting his piece on any vacant cell? The question was first answered by the Dutch mathematician Frederik Schuh in his book *Wonderlijke*

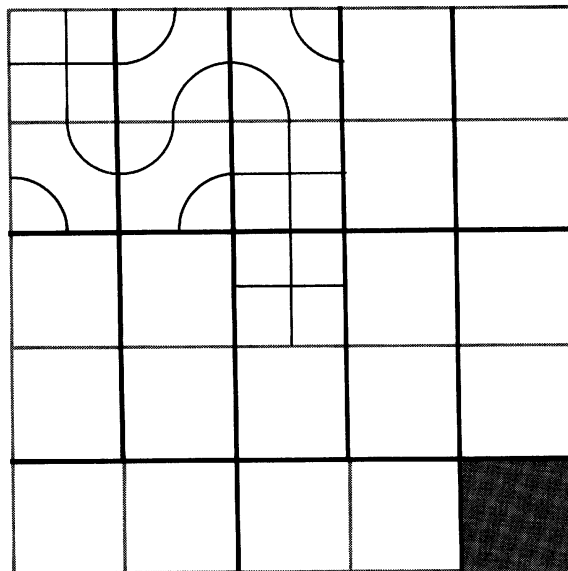
Problemen, published in Holland in 1943. White can always trap Black if he plays rationally. A complete analysis cannot be given here, but the following table shows White's winning responses to Black's six different opening plays.

Black	White
2	A 3 5
4 (or 6)	A 1 5 (or A 3 5)
5	1 2 3
7 (or 9)	A 1 5 (or A 3 5)
8	A 1 5
B	1 2 3

For a complete analysis of the game see F. Gobel's translation of Schuh's book, *The Master Book of Mathematical Recreations*, edited by T. H. O'Beirne (New York: Dover, 1968; pages 239–244). Schuh also analyzes variants of the game. For a good suggestion on how to program a computer to play the game see Donald E. Knuth's *Fundamental Algorithms* (New York: Addison-Wesley, 1968; page 546). Richard Sites, a computer scientist at Stanford University, proved in 1970 that White, regardless of where Black starts, can always trap Black on the board's B cell.

The topological game of Black is won on square boards by the first player if the total number of cells is odd, by the second player if the number of cells is even.

When the play is on an odd-celled board, say a five-by-five, the first player's strategy is to suppose the board, except for the lower right corner cell, is completely covered with dominoes [see Figure 36]. The way the

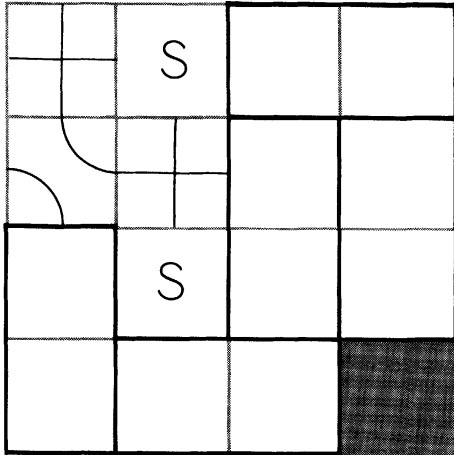


36. Strategy for five-by-five game of Black

dominoes are placed is immaterial. Each move by the second player starts the path on a new domino. The first player then plays so that the path *remains on that domino*. This forces the second player to complete the domino and start the path on another one. It is obvious that the second player eventually will be forced to the border or to an edge of the lower right corner cell.

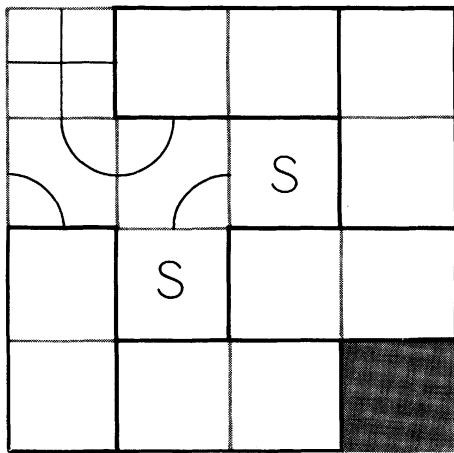
On even-celled square boards the strategy by which the second player wins is more complicated. The board is thought of as being covered with dominoes except for the upper left and lower right corner cells.

Since the two missing cells are the same color, however (supposing that the board is colored like a checkerboard), it is clearly impossible to cover the remaining cells completely with dominoes: there will always be two uncovered cells of the same color. Elwyn R. Berlekamp, who cracked



37. Strategy for four-by-four game of Black

the game, calls these two uncovered cells a “split domino.” The split domino is taken care of by the following clever maneuver: The second player makes his first move as shown in Figure 37, top drawing. This forces the first player to play in the second cell of the main diagonal, and his three possible plays are shown. In each case the unused line of his play will connect two cells of the same color. These two cells, labeled S in the drawings, are regarded as the split domino. The remaining cells (excluding the lower right corner cell) can now be covered with dominoes. Again, the pattern is arbitrary. The second player wins by the domino method previously explained.



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